Qualifying Exam for Ph.D. Candidacy
Department of Physics
October 6th, 2018

Part I

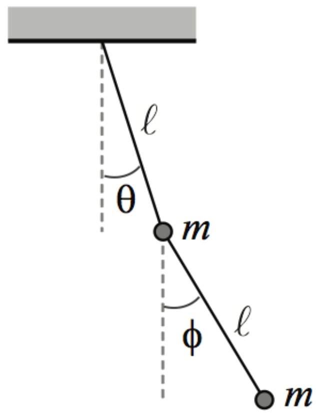
Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants, conversions, etc.:

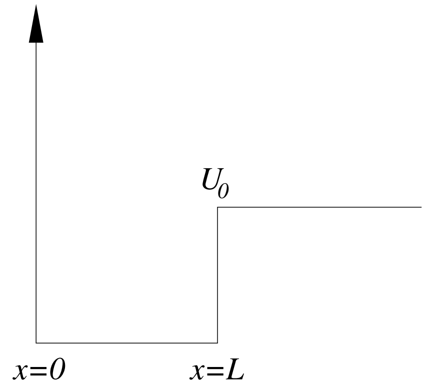
Avogadro's number	N_A	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Gas constant	R	$8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A^{-2}}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^{5}\,\mathrm{N\,m^{-2}}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$
Origin of temperature scales		$0\,^\circ\mathrm{C} = 273\,\mathrm{K}$
1 large calorie (as in nutrition)		$4.184\,\mathrm{kJ}$
1 GeV		$1.609 \times 10^{-10}\,\mathrm{J}$

I–1. Consider the motion of the so-called "double pendulum" consisting of a light inextensible cord of length $2l$ with one end fixed, the other supporting a bob (treated here as a particle) of mass m, and with a second bob, also of mass m, at the center as shown in the figure. Assuming that the system stays in a single plane, we can specify the configuration by two angles, θ and ϕ, as shown. For small oscillations about the equilibrium position, find the normal frequencies of the system.



I–2. Find the particle density as a function of the radial position for a gas of N molecules, each of mass M, in a centrifuge of radius R and length L rotating with angular velocity ω about its axis. Neglect the effects of gravity and assume that the centrifuge has been rotating long enough for the gas of particles to reach equilibrium.

I–3. Prove that, for arbitrarily small values of U_0, the half-infinite well shown in the figure cannot support bound states.

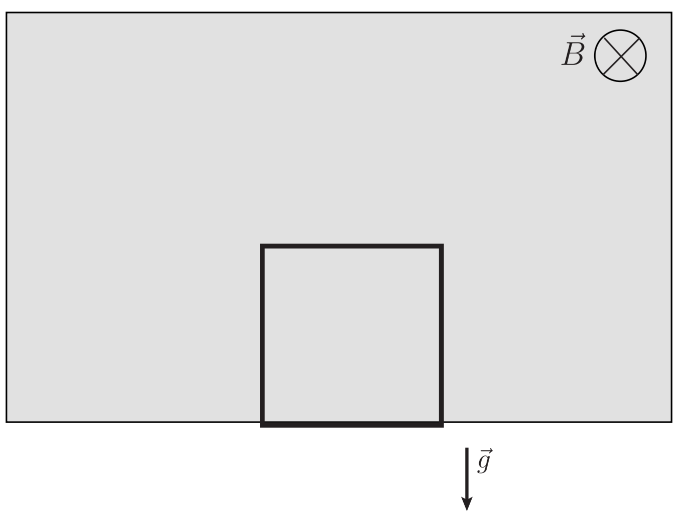


I–4. A conducting square loop with side a, resistance R and total mass m is placed in the uniform magnetic field $\vec{B}$ pointing into the page. Its lower side is at the edge of magnetic field, see figure below. The loop is then released and allowed to fall under gravity.

(a) Which way the induced current will flow in the wire?

(b) Find the velocity of the loop as a function of time.

(c) What happens to the motion of the loop if it is cut?



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Instructions:

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II–1. Two particles of masses m_1 and m_2 interact with potential energy $U = k|\vec{r}_1 - \vec{r}_2|$. The system has angular momentum $\vec{L} = l\hat{z}$ in the center-of-mass (CM) frame. Express your answers in terms of m_1, m_2, k, and l.

(a) What is the radius $r_0 = |\vec{r}_1 - \vec{r}_2|$ of circular orbits?

(b) What is the period T of circular orbits?

(c) Suppose that the circular orbit is slightly perturbed. What is the frequency of small radial oscillations about the circular orbit?

II–2. Consider a gas of n_g atoms in thermal and diffusive equilibrium with N_a adsorption centers each of which has binding energy ϵ_0 and a maximum occupancy of one atom. Let n_s denote the equilibrium number of adsorbed atoms; the total number of atoms in the system N is fixed (i.e. $n_s + n_g = N$).

(a) By constructing the grand partition function for the N_a adsorption centers, show that n_s is given by:

$$n_s = \frac{N_a e^{\mu/\tau} e^{\epsilon_0/\tau}}{1 + e^{\mu/\tau} e^{\epsilon_0/\tau}} \ ,$$

where μ is the chemical potential and $\tau = k_B T$ where T is the temperature and k_B is Boltzmann's constant.

(b) Assuming that the gas is ideal, calculate and sketch the fraction of occupied adsorption centers, n_s/N_a, as a function of temperature in the limit $N_a \ll N$.

(c) Repeat part (b) for the case where $N_a \gg N$.

II–3. A particle of mass m is bound in an attractive delta-function potential

$$V(x) = -\frac{\hbar^2 g}{2m} \delta(x) \ .$$

Its bound-state wave function is $\psi_0(x) = \sqrt{\kappa} e^{-\kappa|x|}$ and its energy is $E_0 = -\hbar^2\kappa^2/(2m)$ with $\kappa = g^2/2$.

(a) Compute the expectation value of the momentum in this state.

Assume that, instantaneously, a force acts on the particle, imparting momentum p_0 to it.

(b) Using the fact that the operator $\exp(p_0\partial/\partial p)$ shifts the momentum of a state by p_0,

$$e^{p_0\frac{\partial}{\partial p}} f(p) = f(p+p_0) \ ,$$

show that the wave function of the new state is

$$\psi(x,p_0) = e^{ip_0x}\psi_0(x) \ .$$

(c) Compute the expectation value of the momentum in the new state.

(d) Calculate the probability of finding the particle in its original state, $\psi_0(x)$.

II–4. A circuit loop contains an ideal voltage signal generator (zero internal impedance), a capacitor with capacitance C and an inductor with inductance L all connected in series. Before time $t = 0$, there is no voltage or current anywhere in the circuit. The capacitor is uncharged. At time $t = 0$, a delta-function voltage pulse is generated,

$$V(t) = V_0\delta(t) \ .$$

(a) Write the differential equation for the charge on the capacitor, $q(t)$ and the current $\dot{q}(t)$.

(b) Determine the charge $q(0_+)$ and the current $\dot{q}(0_+)$. Remember that they are both zero before the pulse. You may assume that the current is always finite.

(c) Determine the current for positive times $q(t)$.