

Candidacy Exam  
Department of Physics  
October 2, 2010

Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants, conversions, etc.:

Avogadro's number	$N_A$	$6.022 \times 10^{23} \text{ mol}^{-1}$
Boltzmann's constant	$k_B$	$1.381 \times 10^{-23} \text{ J K}^{-1}$
Electron charge magnitude	$e$	$1.602 \times 10^{-19} \text{ C}$
Gas constant	$R$	$8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
Planck's constant	$h$ $\hbar = h/2\pi$	$6.626 \times 10^{-34} \text{ J s}$ $1.055 \times 10^{-34} \text{ J s}$
Speed of light in vacuum	$c$	$2.998 \times 10^8 \text{ m s}^{-1}$
Permittivity constant	$\epsilon_0$	$8.854 \times 10^{-12} \text{ F m}^{-1}$
Permeability constant	$\mu_0$	$1.257 \times 10^{-6} \text{ N A}^{-2}$
Gravitational constant	$G$	$6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^5 \text{ N m}^{-2}$
Stefan-Boltzmann constant	$\sigma$	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Electron rest mass	$m_e$	$9.109 \times 10^{-31} \text{ kg} = 0.5110 \text{ MeV } c^{-2}$
Proton rest mass	$m_p$	$1.673 \times 10^{-27} \text{ kg} = 938.3 \text{ MeV } c^{-2}$
Origin of temperature scales		$0^\circ\text{C} = 273 \text{ K}$
1 large calorie (as in nutrition)		4.184 kJ
1 inch		2.54 cm

Definite integrals:

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}. \quad (\text{I-1})$$

$$\int_0^{\infty} x^n e^{-x} dx = \Gamma(n+1) = n!. \quad (\text{I-2})$$

Laplacian in spherical polar coordinates  $(r, \theta, \phi)$ :

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}. \quad (\text{I-3})$$

Laplacian in cylindrical coordinates  $(r, \theta, z)$ :

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}. \quad (\text{I-4})$$

I-1. (a) A rope of length  $l$  is hanging over the edge of a table such that the vertically hanging piece has length  $x_0 < l$ . When released, the rope starts to slide down. Determine the distance  $x(t)$  of the bottom end of the hanging piece to the table top as a function of time, ignoring friction and the resistance of the rope to bending.

(b) Consider the situation in part (i), but now taking into account a friction force which is proportional to the weight of the horizontal piece of the rope lying on the table, with proportionality constant  $\mu$ . Determine  $x(t)$  for initial values  $x(0) = x_0$  with  $0 < x_0 < l$  and  $\dot{x}(0) = v_0 > 0$  at all times with positive  $\dot{x}$ .

I-2. An infinitely long solenoid cylinder extends from  $z = -\infty$  to  $z = +\infty$  and is concentric around the  $z$  axis. The cylinder radius is  $r$ , and it is wrapped with turns of wire that carry current  $I$ . The part of the solenoid in the region  $z < 0$  has  $n$  turns of wire per unit length, while for  $z > 0$  there are  $2n$  turns per unit length. The magnetic field direction along the  $z$  axis is everywhere in the  $+z$  direction.

An electron of mass  $m$ , charge  $e$ , and spin  $\mathbf{s} = \frac{\hbar}{2} \hat{\mathbf{z}}$ , is initially moving in the  $+z$  direction inside the cylinder. For  $z \ll 0$  the electron moves along the  $z$  axis with velocity  $\mathbf{V}_i = v_i \hat{\mathbf{z}}$ . Traveling into region  $z \gg 0$ , the electron velocity changes to  $\mathbf{V}_f = v_f \hat{\mathbf{z}}$ . (Approximate the electron magnetic moment with  $\boldsymbol{\mu} = -\frac{e}{m} \mathbf{s}$ .)

(a) Determine an expression for the magnetic field in the interior of each solenoid.

(b) What is the force on an electron located along the axis inside the solenoid, for  $z \ll 0$ , where the field is uniform?

- (c) Assuming no spin rotation and non-relativistic kinematics, determine the final speed  $v_f$  for  $z \gg 0$  in terms of  $v_i$ ,  $r$ ,  $n$ ,  $I$ ,  $m$ , and  $e$ .

I-3. In an example related to black body radiation, assume that the number of photon modes per unit angular frequency  $\omega$  in an empty cubic box of side  $L$  is

$$\frac{dN}{d\omega} = \frac{\omega^2 L^3}{\pi^2 c^3}. \quad (\text{I-5})$$

If each of these modes is treated as a photon mode that can be populated with an integer number of photons, the energies for a mode will be restricted to  $E = n\hbar\omega$ , where  $n$  is a non-negative integer  $\{0, 1, 2, 3, \dots\}$ .

- (a) Calculate the partition function for the set of modes at one particular angular frequency  $\omega$ . (Be sure to sum the series.)
- (b) Use the partition function to determine the average energy for the modes at angular frequency  $\omega$  at temperature  $T$ .
- (c) Determine an expression for the total electromagnetic energy in the box. (Set up the integral over frequency but don't integrate it).
- (d) Without actually integrating over frequency, it is clear that the energy density will be proportional to  $T^\alpha$ . Find  $\alpha$ .

I-4. A quantum mechanical particle of mass  $m$  is constrained to move between two concentric impermeable spheres of radii  $r = a$  and  $r = b$ . There is no other potential. Find the ground state energy and the corresponding normalized wave function.

Candidacy Exam  
Department of Physics  
October 2, 2010

Part II

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants, conversions, etc.:

Avogadro's number	$N_A$	$6.022 \times 10^{23} \text{ mol}^{-1}$
Boltzmann's constant	$k_B$	$1.381 \times 10^{-23} \text{ J K}^{-1}$
Electron charge magnitude	$e$	$1.602 \times 10^{-19} \text{ C}$
Gas constant	$R$	$8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
Planck's constant	$h$ $\hbar = h/2\pi$	$6.626 \times 10^{-34} \text{ J s}$ $1.055 \times 10^{-34} \text{ J s}$
Speed of light in vacuum	$c$	$2.998 \times 10^8 \text{ m s}^{-1}$
Permittivity constant	$\epsilon_0$	$8.854 \times 10^{-12} \text{ F m}^{-1}$
Permeability constant	$\mu_0$	$1.257 \times 10^{-6} \text{ N A}^{-2}$
Gravitational constant	$G$	$6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^5 \text{ N m}^{-2}$
Stefan-Boltzmann constant	$\sigma$	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Electron rest mass	$m_e$	$9.109 \times 10^{-31} \text{ kg} = 0.5110 \text{ MeV } c^{-2}$
Proton rest mass	$m_p$	$1.673 \times 10^{-27} \text{ kg} = 938.3 \text{ MeV } c^{-2}$
Origin of temperature scales		$0^\circ\text{C} = 273 \text{ K}$
1 large calorie (as in nutrition)		4.184 kJ
1 inch		2.54 cm

Definite integrals:

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}. \quad (\text{II-1})$$

$$\int_0^{\infty} x^n e^{-x} dx = \Gamma(n+1) = n!. \quad (\text{II-2})$$

Laplacian in spherical polar coordinates  $(r, \theta, \phi)$ :

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}. \quad (\text{II-3})$$

Laplacian in cylindrical coordinates  $(r, \theta, z)$ :

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}. \quad (\text{II-4})$$

II-1. Consider two different relativistic proton-proton scattering experiments. In the first experiment a beam of protons is accelerated to energy  $2E$  and allowed to collide with a target (e.g., liquid hydrogen) at rest in the laboratory. In the second, two beams of protons are each accelerated to energy  $E$  and then collided head-on in the laboratory.

- (a) Evaluate the collisional energy in the center-of-mass frame for each experiment. Which experiment allows for more energetic collisions?
- (b) What is the relationship between the energy of the beam in the first experiment, and the energy of each of the beams in the second experiment, that is required for the two experiments to probe the same energy scale? The LHC will allow for 14 TeV center-of-mass energy proton-proton collisions: What energy would the beam in a stationary target experiment need to be to probe the same energy scale?

II-2. Consider a grounded conducting sphere of radius  $a$  the center of which coincides with the origin of the coordinate system. Place a point charge  $q$  at position  $\mathbf{y}$  outside it.

- (a) Find the value  $q'$  and the position  $\mathbf{y}'$  of the image charge inside the sphere.
- (b) Evaluate the potential  $\phi$  at any point  $\mathbf{x}$  outside the sphere.
- (c) Evaluate the surface charge density  $\sigma$  induced on the surface of the sphere.

II-3. Consider a collection of identical flat disks, confined to the  $xy$ -plane, each rotating freely about an axis through its center, i.e., around the  $z$  axis. Let

the disks have moment of inertia  $I$ . Suppose we now treat this as a two-dimensional gas, the disks colliding with each other and transferring rotational kinetic energy such that the angular velocities  $\omega$  vary over a range  $-\infty$  to  $+\infty$ . Determine that the average angular momentum at some equilibrium temperature  $T$  has the form

$$\langle I|\omega| \rangle = \sqrt{\frac{2Ik_B T}{\pi}}. \quad (\text{II-5})$$

II-4. A system has five orbital states into which one can put three identical particles. How many different arrangements are there for:

- (a) spin-zero bosons?
- (b) spin one-half fermions?
- (c) classical particles?