Qualifying Exam for Ph.D. Candidacy
Department of Physics
February 3rd, 2018

Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants, conversions, etc.:

Avogadro's number	N_A	$6.022 \times 10^{23}\,\mathrm{mol}^{-1}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K}^{-1}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Gas constant	R	$8.314\,\mathrm{J\,mol}^{-1}\,\mathrm{K}^{-1}$
Planck's constant	h $\hbar = h/2\pi$	$6.626 \times 10^{-34}\,\mathrm{J\,s}$ $1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s}^{-1}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m}^{-1}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A}^{-2}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^{5}\,\mathrm{N\,m}^{-2}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m}^{-2}\,\mathrm{K}^{-4}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$
Origin of temperature scales		$0\,^\circ\mathrm{C} = 273\,\mathrm{K}$
1 large calorie (as in nutrition)		$4.184\,\mathrm{kJ}$
1 GeV		$1.609 \times 10^{-10}\,\mathrm{J}$

Definite integrals:

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}. \tag{I–1}$$

$$\int_0^\infty x^n e^{-x} dx = \Gamma(n+1) = n!. \tag{I–2}$$

Indefinite integrals:

$$\int \frac{x}{(x^2+a^2)^n} dx = \frac{1}{2(1-n)} \frac{1}{(x^2+a^2)^{n-1}} n + c \quad \text{for } n \neq 0, 1 \tag{I–3}$$

I–1. A J/ψ particle with mass $M = 3.1\,\mathrm{GeV/c^2}$ can decay into a two-particle state consisting of an electron and a positron. In one observation, the electron and positron both have energies equal to $10\,\mathrm{GeV}$. Determine the angle between the electron and the positron in this case. You may assume that on the scale of this problem the electron and positron are massless.

I–2. A ${}^7N_{14}$ nucleus has unit nuclear spin, $I = 1$. Assume that the diatomic nitrogen molecule N_2 can rotate but does not vibrate at ordinary temperatures and ignore electron motion.

(i) Find the relative abundances of the ortho and para-molecules in a sample of nitrogen gas in equilibrium; assume there are no interactions between molecules. (Ortho molecules are in a symmetric spin state; Para molecules are in an antisymmetric spin state)

(ii) What happens to the relative abundances as the temperature is lowered towards absolute zero?

I–3. Consider an electron in a uniform magnetic field along the z direction. Let the result of a measurement be such that the electron spin is along the positive y direction at time $t = 0$. Find the state vector for the spin and the average polarization along the x direction (i.e. the expectation value of S_x) for $t > 0$.

I–4. A metal sphere with radius R and charge Q is surrounded by an inhomogeneous insulator, whose permittivity is given by

$$\epsilon(r, \vartheta, \varphi) = \begin{cases} \epsilon_1(r) & \text{if } 0 \le \vartheta < \delta \\ 2\epsilon_1(r) & \text{if } \delta \le \vartheta \le \pi \end{cases}$$

with a constant δ and $\epsilon_1(r) = r/(r - R/2)$.

Compute the electric potential $\phi(r)$, assuming that it is spherically symmetric, and the charge density on the metal sphere.

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Definite integrals:

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}. \tag{II–1}$$

$$\int_0^\infty x^n e^{-x} dx = \Gamma(n+1) = n!. \tag{II–2}$$

Indefinite integrals:

$$\int \frac{x}{(x^2+a^2)^n} dx = \frac{1}{2(1-n)} \frac{1}{(x^2+a^2)^{n-1}} n + c \quad \text{for } n \neq 0, 1 \tag{II–3}$$

II–1. A charged ball with radius R, mass m_0 and charge q is initially at rest, surrounded by an uncharged cloud of dust with constant mass density ρ. At time $t = 0$, a constant electric field $\vec{E} = E_0 \vec{e}_x$ is turned on. The ball then adsorbs all the dust it encounters in the volume it sweeps out once it starts moving.

Compute the mass $m(t)$ and the velocity $v(t)$ at some time $t > 0$. Assume that at this time the thickness of the surface layer of dust remains much smaller than R and that the density of the dust cloud does not change with time.

II–2. The molecules of an ideal gas have a root mean square speed of 500 m/s and a mean free path of 5×10^{-4} cm.

(i) What is the most probable speed of the molecules?

(ii) What is the mean number of collisions per second that occur between the molecules in the gas?

II–3. Two identical bosons move in the one-dimensional simple harmonic oscillator potential

$$V = \frac{1}{2} m\omega^2 (x_1^2 + x_2^2) \ ;$$

they also interact with each other via the potential

$$V_I = V_0 \exp(-\alpha(x_1 - x_2)^2) \quad \alpha > 0 \ .$$

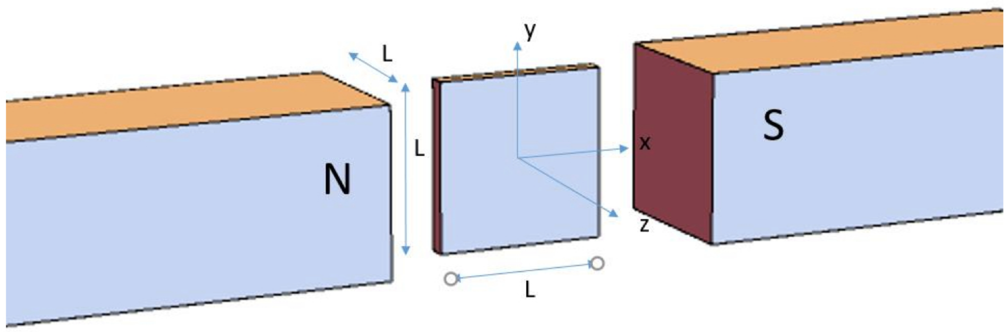
(i) Using that the first order correction to the ground state energy of a non-interacting system due to interactions is given by the expectation value of the interaction Hamiltonian in the ground state, find the ground state energy of this system to first order in V_0.

(ii) Suppose that instead of bosons we now have two identical fermions. Find the ground state energy to first order in V_0 for the singlet spin case.

(iii) Find the ground state energy to first order in V_0 for the triplet spin case.

II–4. A sound wave drives a square ribbon in a magnetic field. A conducting but non-magnetic thin rigid square ribbon is supported between the poles of a magnet with the plane of the ribbon the $x-y$ plane. The thin square ribbon is centered at the origin with corners of the ribbon located at $\{\pm L/2, \pm L/2, 0\}$.

The poles are arranged along the x axis. The surfaces of poles of the magnet are square (dimensions $L \times L$) with sides parallel to the y and z directions. Assume the magnetic field between poles is uniform with magnitude B_0 . The mass per area of the square ribbon is σ.



If the air pressure is of the local sound wave is

$$p(z,t) = \begin{cases} p_1 + p_0 \cos(kz - \omega t) & z < 0 \\ p_1 & z > 0 \text{ but near } z = 0 \end{cases}$$

(i) Determine the displacement of the rigid ribbon along the z axis as a function of time (assume the time average z location is zero and time average speed is zero).

(ii) Determine the emf across the ribbon from top to bottom as a function of time. That is, what is the emf between edges of the ribbon at $y = +L/2$ and $y = -L/2$.